

Exercise 32

Evaluate the limit, if it exists.

$$\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

Solution

Rewrite the numerator, cancel h , and then evaluate the limit.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} &= \lim_{h \rightarrow 0} \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h} = \lim_{h \rightarrow 0} \frac{x^2 - (x^2 + 2xh + h^2)}{x^2h(x+h)^2} = \lim_{h \rightarrow 0} \frac{-2xh - h^2}{x^2h(x+h)^2} \\ &= \lim_{h \rightarrow 0} \frac{-2x - h}{x^2(x+h)^2} \\ &= \frac{-2x}{x^2(x^2)} \\ &= -\frac{2}{x^3} \end{aligned}$$